Conformality and Brodsky

Paul H. Frampton [1]

Department of Physics and Astronomy, University of North Carolina, Chapel Hill, NC 27599-3255, USA.

Abstract. In this article I describe the recently-conjectured field-string duality which suggests a class of nonsupersymmetric gauge theories which are conformal (CGT) to leading order of 1/N and some of which may be conformal for finite N. If the standard model becomes conformal at TeV scales, model-building on this basis is an interesting direction. Some remarks are added about the inspiring career of Stan Brodsky.

1. Introductory remark

It is irresistible to take advantage of a typographical error in the printed program for this meeting where my title was off by two letters: "Conformity and Brodsky". Of course I would be very pleased if in ten years time at Brodsky's 70th birthday conformality had become conformity! As will become clear below conformality is antithetical to the presently most popular ideas for extending the standard model, *viz.* grand unification and lessens the motivation for another popular idea, *viz.* low-energy supersymmetry.

2. Abbreviated History of String Theory

The recent development of field string duality possesses some quality of *déja vu* and yet seems the most promising development in the theory in terms of its most optimistic prognosis that it may provide a successful connection between string theory and the real world, and in doing so necessarily a first connection between gravity and the other interactions.

[1] E-mail: frampton@physics.unc.edu

The initial seed of string theory was the Veneziano model in 1968[1]. At the time, finite energy and superconvergence sum rules for hadron scattering (the subject of my DPhil thesis) posed a "duality" of descriptions generally similar to the now-proposed one between a ten-dimensional superstring (or 11 dimensional M theory) and a conformal gauge theory. The hadron sum rules equated quantities of quite different functional dependences on the Mandelstam variables s and t, seemingly an impossibility until the Veneziano model showed an explicit realization.

From 1968 to 1973 the resultant dual resonance models were leading candidates to describe the strong interactions. In 1973 they were, however, elbowed aside by an alternative theory, quantum chromodynamics (QCD). Now the discarded theory is dual to the QCD which replaced it![2, 3]. This is what I mean by *déja vu*.

The decade 1974-1984 saw a hiatus in string theory. In 1984-85 the First Superstring Revolution included a stab at nearly a Theory of Everything, the perturbative $E(8) \times E(8)$ heterotic string[4]. But its apparent uniqueness turned out to be illusory (as was its perturbativity), and another decade 1985-95 of quiescence followed.

In 1995 came the Second Superstring evolution, and in 1997 the $2\frac{1}{2}$-revolution with AdS/CFT duality. Understanding of duality between weak and strong coupling of supersymmetric field theories led to a corresponding breakthrough in string theory culminating with the idea of M theory as a more basic theory which unified all of the five known ten-dimensional superstings (Types I, IIA, IIB and the O_{32} and $E_8 \times E_8$ heterotic strings) as well as eleven-dimensional supergravity by duality transformations.

One of the most important realizations of the recent period is that string solitons, or D branes, play a dynamical role in the theory equally as important as do the superstrings themselves. D branes are crucial for the field-string duality which is our principal subject. The string duality has also led to a better understanding of the quantum mechanics of black holes.

Our starting point here is the duality between string theory in 10 dimensions (or of M theory in 11 dimensions) and gauge field theory in four dimensions. As already mentioned, this in a real sense closes a 25-year cycle in the history of strings.

Certainly one of the major changes in string theory in the recent years is the appreciation of the role of D branes which are topological defects on which open strings can end. Their necessity in string theory was realized only in 1989[5] and particularly in 1995[6]. Their presence follows from considering the $R \to 0$ limit of a bosonic string compactified on a circle of radius R. Open strings, unlike the closed strings which are necessarily contained in the same theory, cannot wrap around the compactified dimension in the $R \to 0$ limit. Hence for a consistent theory the open strings do not simply end, but are attached to D branes.

These D branes have their own dynamics and play a central role in the full non-perturbative theory. D branes have provided insight into (1) Black hole quantum mechanics; (2) Large N gauge field theory (discussed here).

In general, the term duality applies to a situation where two quite different descriptions are available for the same physics.

The difference can be very striking. For example, in 1997 Maldacena[2] proposed the duality between $d = 4$ $SU(N)$ gauge field theory (GFT) and a $d = 10$ superstring. In the perturbative regime of the GFT this duality cannot hold just because the degrees of freedom are missing, but non-perturbatively the GFT contains sufficient additional states at strong coupling for the duality to be indeed possible.

Take a Type IIB superstring (closed, chiral) in $d = 10$ and compactify it on the manifold:

$$(AdS)_5 \times S^5 \tag{1}$$

Here $(AdS)_5$ is a 5-dimensional Anti De Sitter space whose four dimensional surface M_4 is the $d = 4$ spacetime in which the $SU(N)$ GFT occurs. Note that the isometry group of $(AdS)_5$ is $SO(4, 2)$, the conformal group for four dimensional spacetime. The S^5 is a five-sphere with isometry $SU(4)$ which is the R symmetry of the resultant $\mathcal{N} = 4$ supersymmetric $SU(N)$ GFT. The S^5 can be regarded as a surface in a C_3 three-dimensional space in which N D3 branes are coincident.

The D branes each carry an associated $U(1)$ gauge symmetry. This is understandable as a correct generalization of the Chan-Paton factors[7] which were once used to attach charges to the ends of open strings. N parallel D branes with vanishing separation yield a $U(N)$ gauge group where the additional $N^2 - N$ gauge bosons arise from connecting open strings which become massless in the zero-length limit. This $U(N)$ turns out to be broken to $SU(N)$ by the brane dynamics. The resultant $\mathcal{N} = 4$ SUSY Yang-Mills theory is well-known to be a very well-behaved, finite field theory. It is conformally invariant even for finite N with all RG $\beta-$ functions (gauge, Yukawa and quartic Higgs) vanishing.

This perturbative finiteness was proved in 1983 by Mandelstam[8]. The Maldacena conjecture is primarily aimed at the $N \to \infty$ limit with the 't Hooft parameter[9] of N times the squared gauge coupling fixed, and makes no claim concerning conformality for finite N[10]. But since the $\mathcal{N} = 4$ case is known to be conformal even for finite N one is tempted to extend the conjecture[11] to finite N cases even where all supersymmetry is broken. In that case the standard model can be a part of a conformal nonsupersymmetric gauge theory where the $\beta-$ functions become zero at a TeV scale. Then the coupling constants cease to run and there is no grand unification. This nullifies the gauge hierarchy problem between the weak scale and the GUT scale, and yet it is still possible to derive the correct electroweak mixing angle[12]. In particular there is no reason to invoke low-energy supersymmetry either. Gravity is itself non-conformal (it necessitates the dimensionful Newton constant). We shall address this at the end of the article.

3. Breaking Supersymmetries.

To approach the real world one needs less supersymmetry than $\mathcal{N} = 4$, in fact the empirical data presently suggest no supersymmetry at all, $\mathcal{N} = 0$.

By factoring out a discrete group (we shall assume it is an abelian discrete group, only because that case has been most fully investigated; it is possible that a non-abelian discrete group can work as well) and composing the orbifold:

$$S^5/\Gamma \tag{2}$$

one may break $\mathcal{N} = 4$ supersymmetry to $\mathcal{N} = 2, 1$ or 0. Of special interest is the $\mathcal{N} = 0$ case.

We may take $\Gamma = Z_p$ which identifies p points in C_3.

The rule for breaking the $\mathcal{N} = 4$ supersymmetry is:

$$\Gamma \subset SU(2) \Longrightarrow \mathcal{N} = 2 \tag{3}$$

$$\Gamma \subset SU(3) \Longrightarrow \mathcal{N} = 1 \tag{4}$$

$$\Gamma \not\subset SU(3) \Longrightarrow \mathcal{N} = 0 \tag{5}$$

In fact, to specify the embedding of $\Gamma = Z_p$, we need to identify three integers $a_i = (a_1, a_2, a_3)$ such that the action of Z_p on C_3 is:

$$C_3 : (X_1, X_2, X_3) \xrightarrow{Z_p} (\alpha^{a_1} X_1, \alpha^{a_2} X_2, \alpha^{a_3} X_3) \tag{6}$$

with

$$\alpha = exp\left(\frac{2\pi i}{p}\right) \tag{7}$$

The scalar multiplet is in the **6** of $SU(4)$ R symmetry and is transformed by the Z_p transformation:

$$diag(\alpha^{a_1}, \alpha^{a_2}, \alpha^{a_3}, \alpha^{-a_1}, \alpha^{-a_2}, \alpha^{-a_3}) \tag{8}$$

together with the gauge transformation

$$diag(\alpha^0, \alpha^1, \alpha^2, \alpha^3, \alpha^4, \alpha^5) \quad \times \quad \alpha^i \tag{9}$$

for the different $SU(N)_i$ of the gauge group $SU(N)^p$.

What will be relevant are states invariant under a combination of these two transformations, as discussed in the next subsection.

If $a_1 + a_2 + a_3 = 0 \pmod{p}$ then the matrix

$$\begin{pmatrix} a_1 & & \\ & a_2 & \\ & & a_3 \end{pmatrix} \tag{10}$$

is in $SU(3)$ and hence $\mathcal{N} \geq 1$ is unbroken and this condition must therefore be avoided if we want $\mathcal{N} = 0$.

If we examine the **4** of $SU(4)$, we find that the matter which is invariant under the combination of the Z_p and an $SU(N)^p$ gauge transformation can be deduced similarly.

It is worth defining the spinor **4** explicitly by $A_q = (A_1, A_2, A_3, A_4)$ with the A_q, like the a_i, defined only mod p. Explicitly we may define $a_1 = A_1 + A_2$, $a_2 = A_2 + A_3$, $a_3 = A_3 + A_1$ and $A_4 = -(A_1 + A_2 + A_3)$. In other words, $A_1 = \frac{1}{2}(a_1 - a_2 + a_3)$, $A_2 = \frac{1}{2}(a_1 + a_2 - a_3)$, $A_3 = \frac{1}{2}(-a_1 + a_2 + a_3)$, and $A_4 = -\frac{1}{2}(a_1 + a_2 + a_3)$. To leave no unbroken supersymmetry we must obviously require that all A_q are non-vanishing. In terms of the a_i this condition which we shall impose is:

$$\sum_{i=1}^{i=3} \pm(a_i) \neq 0 \pmod{p} \tag{11}$$

The question at issue is whether the gauge theories derived in this way are conformal for finite N. What is known is that at leading order in $1/N$ the $\beta-$ functions vanish to all orders in perturbation theory[13]. This is already remarkable from the field theory point of view because without the stimulus of the AdS/CFT duality it would be difficult to guess any $\mathcal{N} = 0$ theory with all $\beta-$ functions zero to leading order in $1/N$ and all orders in the GFT coupling. Without non-renormalization theorems this imposes an infinite number of constraints on a finite number of choices of the fermion and scalar representations of $SU(N)^p$.

Nevertheless, since $\mathcal{N} = 4$ is conformal (all $\beta-$ functions vanish) we can be more ambitious and ask[11] that all $\beta-$ functions vanish even for finite N, at least for some fixed point in coupling constant space, and use the construction to motivate phenomenological model-building.

4. Matter Representations.

The Z_p group identifies p points in C_3. The N converging D branes approach all p such points giving a gauge group with p factors:

$$SU(N) \times SU(N) \times SU(N) \times .... \times SU(N) \tag{12}$$

The matter which survives is invariant under a product of a gauge transformation and a Z_p transformation.

For the covering gauge group $SU(pN)$, the transformation is:

$$(1, 1, ..., 1; \alpha, \alpha, ...\alpha; \alpha^2, \alpha^2, ...\alpha^2; ......; \alpha^{p-1}, \alpha^{p-1}, ...\alpha^{p-1}) \qquad (13)$$

with each entry occurring N times.

Under the Z_p transformation for the scalar fields, the **6** of $SU(4)$, the tranformation is

$$\sim \quad \underline{X} \Rightarrow \quad (\alpha^{a_1}, \alpha^{a_2}, \alpha^{a_3}) \qquad (14)$$

The result can conveniently be summarized by a *quiver diagram*[14]. One draws p points and for each a_k one draws a non-directed arrow between all modes i and $i+a_k$. Each arrow denotes a bi-fundamental representation such that the resultant scalar representation is:

$$\sum_{k=1}^{k=3}\sum_{i=1}^{i=p}(N_i, \bar{N}_{i\pm a_k}) \qquad (15)$$

If $a_k = 0$ the bifundamental is to be reinterpreted as an adjoint representation plus a singlet representation.

For the chiral fermions one must construct the spinor **4** of $SU(4)$. The components are the A_q given above. The resultant fermion representation follows from a different quiver diagram. One draws p points and connects with a *directed* arrow the node i to the node $i + A_q$. The fermion represntation is then:

$$\sum_{q=1}^{q=4}\sum_{i=1}^{i=p}(N_i, \bar{N}_{i+A_q}) \qquad (16)$$

Since all $A_q \neq 0$, there are no adjoint representations for fermions. This completes the matter representation of $SU(N)^p$.

We have begun the selection process between these models by looking at one and two loop $\beta-$functions. At one loop we are still at leading order in N at least for β_g so there is coincidence with the $\mathcal{N} = 4$ case. At 2 loops we found[15] already that only 8% of a sample satisfy one criterion, the fraction remaining alive diminishing like $3/p$ for large p.

Checking the Yukawa and Higgs running for 2 loops needs more calculation of couplings and is underway.

Beyond that:

- If all 2-loop tests are satisfied, what about 3 or more? It rapidly becomes impractical to take the approach of direct calculation.

- There is the question of uniqueness of any surviving $\mathcal{N} = 0$ CGT.

- The CGT may be inspirational in model building, to be discussed below.

Why $\mathcal{N} = 0$?.

$\mathcal{N} = 1$ is motivated by accommodation of the gauge hierarchy M_{GUT}/M_{WEAK}.

In a conformal gauge theory the gauge couplings cease to run and the GUT scale does not exist; this hierarchy is therefore nullified.

Low-scale Kaluza-Klein[16] is similar to the conformality approach in this particular regard, although the idea is quite different.

More philosophically, we may recall the over 50 years ago the infinite renormalization of QED was greeted with much skepticism. If the conformality of even $\mathcal{N} = 4$ CFT had been already discovered, surely the skepticism would have been far greater?

5. Conformality and Particle Phenomenology.

Let us itemize the following points:

- The hierarchy between the GUT and weak scales is 14 orders of magnitude.
- Why do the two very different scales exist?
- How are the scales stabilized under quantum corrections?
- Supersymmetry solves the second problem but not the first.

Successes of supersymmetry.

- Cancellations of UV infinities.
- Technical naturalness of hierarchy.
- Unification of gauge couplings.
- Natural appearance in string theory.

Puzzles about supersymmetry.

- The “mu” problem - why is the Higgs mass at the weak scale and not at the Planck scale (hierarchy).

- Breaking supersymmetry leads to too large a cosmological constant.
- Is supersymmetry fundamental to string theory?
- There are solutions of string theory without supersymmetry.

Supersymmetry replaced by conformality at TeV scale.
The following aspects of the idea are discussed:

- The idea is possible.
- Explicit examples containing standard model states.
- Finiteness as a more rigid constraint than supersymmetry.
- Predicts additional states for finiteness/conformality.
- Rich inter-family structure of Yukawa couplings.

6. Conformality as Hierarchy Solution.

The quark and lepton masses, the QCD scale and the weak scale are extremely small compared to a TeV scale. They may all be put to zero suggesting: add degrees of freedom to yield GFT with conformal invariance (CGT). 't Hooft's naturalness condition holds since zero mass increases the symmetry.

The theory is assumed to be given by the action[17]

$$S = S_0 + \int d^4x \alpha_i O_i \tag{17}$$

where S_0 is the action for the conformal theory and the O_i are operators with dimension below four which break conformal invariance softly.

The mass parameters α_i have mass dimension $4 - \Delta_i$ where Δ_i is the dimension of O_i at the conformal point.

Let M be the scale set by the parameters α_i and hence the scale at which conformal invariance is broken. Then for $E \gg M$ the couplings will not run while they start running for $E < M$. To solve the hierarchy problem we assume M is close to the TeV scale.

7. Large Class of d=4 QFTs - Each SU(4) Subgroup.

There is first the choice of N. One knows that for leading $1/N$ the theory is conformal. What about finite N? One expects at least a conformal fixed point in some cases. One starts from $\mathcal{N} = 4$ GFT, eliminates fields and re-identifies others such that conformality results.

It is important to realize that, even *without* supersymmetry, boson-fermion number equality holds, and underlies the finiteness.

Let $\Gamma \subset SU(4)$ denote a discrete subgroup of $SU(4)$. Consider irreducible representations of Γ. Suppose there are k irreducible representations R_i, with dimensions d_i with $i = 1, ..., k$. The gauge theory in question has gauge symmetry

$$SU(Nd_1) \times SU(Nd_2) \times ...SU(Nd_k)$$

The fermions in the theory are given as follows. Consider the 4 dimensional representation of Γ induced from its embedding in $SU(4)$. It may or may not be an irreducible representation of Γ. We consider the tensor product of $\mathbf{4}$ with the representations R_i:

$$\mathbf{4} \otimes R_i = \oplus_j n_i^j R_j$$

The chiral fermions are in bifundamental representations

$$(1, 1, .., \mathbf{Nd_i}, 1, ..., \overline{\mathbf{Nd_j}}, 1, ..)$$

with multiplicity n_i^j defined above. For $i = j$ the above is understood in the sense that we obtain n_i^i adjoint fields plus n_i^i neutral fields of $SU(Nd_i)$. Note that we can equivalently view n_i^j as the number of trivial representations in the tensor product

$$(\mathbf{4} \otimes R_i \otimes R_j^*)_{trivial} = n_i^j$$

The asymmetry between i and j is manifest in the above formula. Thus in general we have

$$n_i^j \neq n_j^i$$

and so the theory in question is in general a chiral theory. However if Γ is a real subgroup of $SU(4)$, i.e. if $\mathbf{4} = \mathbf{4}^*$ as far as Γ representations are concerned, then we have by taking the complex conjugate:

$$n_i^j = (\mathbf{4} \otimes R_i \otimes R_j^*)_{trivial} = (\mathbf{4} \otimes R_i^* \otimes R_j)_{trivial} = n_j^i.$$

So the theory is chiral if and only if $\mathbf{4}$ is a complex representation of Γ, i.e. if and only if $\mathbf{4} \neq \mathbf{4}^*$ as a representation of Γ. If Γ were a real subgroup of $SU(4)$ then $n_i^j = n_j^i$.

If Γ is a complex subgroup, the theory is chiral, but it is free of gauge anomalies. To see this, note that the number of chiral fermions in the fundamental representation of each group $SU(Nd_i)$ plus Nd_i times the number of chiral fermions in the adjoint representation is given by

$$\sum_j n_i^j N d_j = 4Nd_i$$

(where the number of adjoints is given by n_i^i). Similarly the number of anti-fundamentals plus Nd_i times the number of adjoints is given by

$$\sum_j n_j^i N d_j = \sum N d_j (4 \otimes R_j \otimes R_i^*)_{trivial} = \sum N d_j (4^* \otimes R_j^* \otimes R_i)_{trivial} = 4Nd_i$$

Thus we see that the difference of the number of chiral fermions in the fundamental and the anti-fundamental representation is zero (note that the adjoint representation is real and does not contribute to anomaly). Thus each gauge group is anomaly free.

In addition to fermions we also have bosons in bi-fundamental representations. The number of bosons M_i^j in the bi-fundamental representation of $SU(Nd_i) \otimes SU(Nd_j)$ is given by the number of R_j representations in the tensor product of the representation **6** of $SU(4)$ restricted to Γ with the R_i representation. Note that since **6** is a real representation we have

$$M_i^j = (6 \otimes R_i \otimes R_j^*)_{trivial} = (6 \otimes R_i^* \otimes R_j)_{trivial} = M_j^i$$

In other words for each M_i^j we have a *complex* scalar field in the corresponding bi-fundamental representation.

Interactions. The interactions of the gauge fields with the matter is fixed by the gauge coupling constants for each gauge group. The inverse coupling constant squared for the i-th group combined with the theta angle for the i-th gauge group is

$$\tau_i = \theta_i + \frac{i}{4\pi g_i^2} = \frac{d_i \tau}{|\Gamma|}$$

where $\tau = \theta + \frac{i}{4\pi g^2}$ is an arbitrary complex parameter independent of the gauge group and $|\Gamma|$ denotes the number of elements in Γ.

There are two other kinds of interactions: Yukawa interactions and quartic scalar field interactions. The Yukawa interactions are in 1-1 correspondence with triangles in the quiver diagram with two directed fermionic edges and one undirected scalar edge, with compatible directions of the fermionic edges:

$$S_{Yukawa} = \frac{1}{g^2} \sum_{directed\ triangles} d^{abc} \mathrm{Tr} \psi_{ij^*}^a \phi_{jk^*}^b \psi_{ki^*}^c$$

where a, b, c denote a degeneracy label of the corresponding fields. d^{abc} are flavor dependent numbers determined by Clebsch-Gordon coefficients as follows: a, b, c determine elements u, v, w (the corresponding trivial representation) in $\mathbf{4} \otimes R_i \otimes R_j^*$, $\mathbf{6} \otimes R_j \otimes R_k^*$ and $\mathbf{4} \otimes R_k \otimes R_i^*$. Then

$$d^{abc} = u \cdot v \cdot w$$

where the product on the right-hand side corresponds to contracting the corresponding representation indices for R_m's with R_m^*'s as well as contracting the $(\mathbf{4} \otimes \mathbf{6} \otimes \mathbf{4})$ according to the unique $SU(4)$ trivial representation in this tensor product.

Similarly the quartic scalar interactions are in 1-1 correspondence with the 4-sided polygons in the quiver diagram, with each edge corresponding to an undirected line. We have

$$S_{Quartic} = \frac{1}{g^2} \sum_{4-gons} f^{abcd} \Phi^a_{ij^*} \Phi^b_{jk^*} \Phi^c_{kl^*} \Phi^d_{li^*}$$

where again the fields correspond to lines a, b, c, d which in turn determine an element in the tensor products of the form $\mathbf{6} \otimes R_m \otimes R_n^*$. f^{abcd} is obtained by contraction of the correponding element as in the case for Yukawa coupling and also using a $[\mu, \nu][\mu, \nu]$ contraction in the $\mathbf{6} \otimes \mathbf{6} \otimes \mathbf{6} \otimes \mathbf{6}$ part of the product.

Conformal Theories in 4 Dimensions. There follows a large list of quantum field theories in 4 dimensions, one for each discrete subgroup of $SU(4)$ and each choice of integer N, motivated from string theory considerations which has been proven to have vanishing beta function to leading order in N. Below we argue for the existence of at least one fixed point even for finite N (under some technical assumptions). The vanishing of the beta function at large N can also be argued using AdS/CFT correspondence.

Consider strong-weak duality. This duality exchanges $8\pi g^2 \leftrightarrow 1/8\pi g^2$ (at $\theta = 0$). This follows from their embedding in the type IIB string theory which enjoys the same symmetry. In fact, this gauge theory *defines* a particular type IIB string theory background and so this symmetry must be true for the gauge theory as well. In the leading order in N the beta function vanishes. Let us assume at the next order there is a negative beta function, i.e., that we have an asymptotically free theory. Then the flow towards the infrared increases the value of the coupling constant. Similarly, by the strong-weak duality, the flow towards the infrared at large values of the coupling constant must decrease the value of the coupling constant. Therefore we conclude that the beta function must have at least one zero for a finite value of g.

This argument is not rigorous for three reasons: One is that we ignored the flow for the θ angle. This can be remedied by using the fact that the moduli space is the upper half-plane modulo $SL(2, Z)$ which gives rise to a sphere topology and using the fact that any vector field has a zero on the sphere ("it is impossible to comb the hair on a sphere"). The second reason is that we assumed asymptotic freedom at the first non-vanishing order in the large N expansion. This can in principle be checked by perturbative techniques and at least it is not a far-fetched assumption. More serious, however, is the assumption that there is effectively one coupling constant. It would be interesting to see if one can relax this assumption, which is valid at large N.

8. Comments on Conformality

The most exciting aspect of the conformality approach is in model building beyond the standard model. The reason the model building is so interesting is that not only the fermion but also the scalar representations are prescribed by the construction. Thus one may not simply add whatever Higgs scalars are required for the appropriate symmetry breaking. This rigidity is actually helpful. Lack of adequate space precludes including details of the model building described in[12, 18]. Clearly a simple model would encourage support for this approach. The simplest model using abelian orbifolds and found in [18] is based on the gauge group $SU(3)^7$. This has less generators than the $E(6)$ gauge group and may therefore be of considerable interest. Non-abelian orbifolds are currently under examination.

The final issue concerns gravity. In the CGT for strong and electroweak interactions there is no manifest gravity. One may say there is no evidence for a graviton and that one is concerned only with observable physics. Nevertheless if one extrapolates to extremely high energy gravity should enter and it is *not* conformally invariant because, of course, the Newton constant is dimensional. It would be attractive to understand the incorporation of gravitation while staying in only four spacetime dimensions but this possibility remains elusive. The CGT itself stands on its own without need of the string from which its construction was inferred. But to describe gravity the most promising idea seems to be to add an extra dimension and consider $(AdS)_5$. Keeping the full range of the fifth coordinate leads one back to the absence of gravity on the surface. But, as pointed out in [19], truncating the range of the fifth coordinate leads to a metric field on the surface and hence to a graviton.

As a final speculation, is it possible that conformality is related to the vanishing cosmological constant? Until conformal invariance is broken the vacuum energy is zero. It then depends on how softly conformal invariance can be broken if a $(TeV)^4$ contribution is to be avoided. Clearly, the breaking of conformal invariance needs to be studied, not only for this reason, but also to allow predictions for dimensionless quantities like mass ratios and mixing angles in the low-energy theory.

9. Remarks on Stanley J. Brodsky

This workshop is in honor of Stan Brodsky's 60th birthday. I first met Stan in 1969 at SLAC when he was 29 years old. At that time, and ever since, as H.C. Pauli said here, he has been very welcoming to visitors at SLAC. Stan is more than a sociable chap since, as R. Braun informed me, he has published 326 papers since then, an average of almost one paper every month. As we have heard today these papers have had a very significant impact on the development of quantum electrodynamics and quantum chromodynamics. When asked his favorite of his own papers, Stan selected an article with Lepage: *Exclusive Processes in Perturbative Quantum Chromodynamics* Phys. Rev. **D22,** 2157 (1980).

Stan Brodsky's total work has already attracted well over ten thousand citations. This puts him in an exclusive club for theoretical physicists and represents an inspiring career which I hope will continue for decades to come.

Acknowledgements

Thanks are due to George Strobel for organizing this meeting. This work was supported in part by the US Department of Energy under Grant No. DE-FG02-97ER41036.

References

[1] Veneziano G 1968 Nuov Cim **57A** 190

[2] Maldacena J 1998 Adv Theor Math Phys **2** 231

[3] Aharony O, Gubser S S, Maldacena J, Ooguri H and Oz Y hep-th/9905111

[4] Candelas P, Horowitz G T, Strominger A and Witten E 1985 Nucl Phys **B258** 46

[5] Dai J, Leigh R G and Polchinski J 1989 Mod. Phys. Lett. **A4** 2073

[6] Polchinski J 1995 Phys Rev Lett **75** 4724

[7] Paton and Chan H M 1969 Nucl Phys **B10** 516

[8] Mandelstam S 1983 Nucl Phys **B213** 149

[9] 't Hooft G 1972 Nucl Phys **B72** 461

[10] Maldacena J (private communication)

[11] Frampton P H 1999 Phys Rev **D60** 041901

[12] Frampton P H 1999 hep-th/9905042 Phys Rev **D60** (Oct 15 1999 issue)

[13] Bershadsky M and Johansen A 1998 Nucl Phys **B536** 141

[14] Douglas M and Moore G 1996 hep-th/9603167

[15] Frampton P H and Shively W F 1999 Phys Lett **B454** 49

[16] Antoniadis I 1990 Phys Lett **246** 377

[17] Frampton P H and Vafa C 1999 hep-th/9903226

[18] Frampton P H 1999 hep-th/9907051

[19] Verlinde H 1999 hep-th/9906182